Calculation of Magnetic Moments of Λ-hypernuclei $^{13}_{\Lambda}C$, $^{17}_{\Lambda}O$ and $^{41}_{\Lambda}Ca$

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ABSTRACT

The magnetic moments of Λ-hypernuclei are the most interesting observables, which provide a highly sensitive probe of lambda in the hypernuclei structure and also supply direct information on hyperon-nucleon interactions. In this work, we derive the magnetic moments of Λ-hypernuclei such as $^{13}_{\Lambda}C$, $^{17}_{\Lambda}O$ and $^{41}_{\Lambda}Ca$ employ a relativistic approach in the fact of the Dirac equation and the spin-orbital potential in their ground and excited, i.e., the $1S_{1/2}$, $1P_{3/2}$ and $1P_{1/2}$ conditions. We, then, extract an analytic solution for the wave function of hyperon, which is needed for computing the magnetic moments of Λ-hypernuclei. The hypernuclei magnetic moments are the magnetic moment of the last unpaired baryon for the odd mass hypernuclei; therefore, in our work, we study the hypernuclear magnetic moment with one Lambda added to a closed-shell core of nucleons. Since Λ-hypernuclei is an isoscalar particle, it is possible to probe the modified core current electromagnetically directly.

Keywords: Lambda Hypernuclei, Hyperon, Dirac Magnetic Moment, Anomalous Magnetic Moment.

I. Introductions

Hypernuclei are complicated nuclear systems where one or more hyperons replace one or more nucleons. Hypernuclei can be described by their properties such as the binding energy, electric charge, magnetic moment, etc. Among these quantities, the magnetic moment provides a highly sensitive probe of single-particle structure and serves as a stringent test of nuclear models [1]. It should be noted that nuclei's magnetic moments have also been a driving force of various new ideas in nuclear theory, such as the single-particle picture, the configuration mixing [2], and the meson exchange currents [3]. It is, then, expected that the more accurate study of magnetic moments of hypernuclei may reveal new aspects of Hypernuclear physics. On the other side, the hypernuclei magnetic moments can provide direct information on hyperon-nucleon interaction and the role of hadrons in the nuclear medium [4]. It can further probe the shell structure of the nucleon

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as well as the lambda in hypernuclei [5]. It should be mentioned that the magnetic moment may be sensitive to the spin and angular momentum structure of hypernuclei as well as the spin-dependent hyperon-nucleon interactions. The relativistic and nonrelativistic schemes give similar values for nuclear magnetic moments. As a short description, in the nonrelativistic shell model, the Schmidt values of magnetic moments are obtained directly by neglecting the core polarization due to a particle or a hole added.

On the other hand, in relativistic approaches the results come from the compensation of two effects, the enhancement of the valence particle current due to the reduction of the effective nucleon mass and the contribution of the additional current raised from polarized core nucleons [6]. Applied relativistic models give the magnetic moments of Λ-hypernuclei remarkably different from Schmidt values, as is expected [7]. The main differences are related almost entirely to the Dirac part of magnetic moments because the anomalous part of magnetic moments agrees with Schmidt values up to 0.2% [6]. Due to the neutrality of Λ, the Dirac part of the magnetic moments originates in the nuclear core's polarization. The sign of the magnetic moment is specified by the anomalous position and is negative (positive) for $\ell+1/2\,(\ell-1/2)$ states while the Dirac part and, thus, the core contribution always has a negative amount for the mentioned cases. In this work, in a relativistic approach, we analytically determine the magnetic moment of several Λ-hypernuclei such as $^{13}_{\Lambda}\mathrm{C}$, $^{17}_{\Lambda}\mathrm{O}$ and $^{41}_{\Lambda}\mathrm{Ca}$ in their ground and excited states, i.e. the $1\mathrm{s}_{1/2}$, $1\mathrm{p}_{3/2}$ and $1\mathrm{p}_{1/2}$ states. For our analysis, we consider the spin-orbital potential and determine the wave function of hypernuclei. Then we compute the Dirac, the anomalous and the total

magnetic moments of the ground, and the excited states of Λ-hypernuclei.

II. Research Theories

The Dirac equation for spin-1/2 particles in the natural units (where $\hbar = \mathrm{c} = 1$) is given by

$$\left[\vec{\alpha}\cdot\vec{p}+\beta\left(M+U_S(r)\right)-E+U_V(r)\right]\Psi(r)=0, \tag{1}$$

$$\text{with } \psi(\vec{r})=\frac{1}{r}\begin{pmatrix} F_{nk}(r)Y_{jm}^{\ell}(\theta,\varphi) \\ iG_{nk}(r)Y_{jm}^{\tilde{\ell}}(\theta,\varphi) \end{pmatrix}$$

where, β and $\vec{\alpha}=(\alpha_1,\alpha_2,\alpha_3)$ are the Dirac matrices. p and E refer to the momentum and relativistic energy of particles. F(r) and G(r) are wave functions. "n" is the radial quantum number, and $k(=\pm(j\pm1/2))$ (j total angular-momentum) is the possible eigenvalue of the operator $\hat{K}(=\beta(\hat{\sigma}\cdot\vec{L}+1))$. Here, U_S and U_V are the scalar and the vector potentials, respectively. M is the baryon mass. Putting β and $\vec{\alpha}$ in Eq. (1), two coupled first-order Dirac equations are obtained

$$F_{nk}''(r)+k\,r^{-1}F_{nk}(r)=(M+E-\Delta)G_{nk}(r), \tag{2}$$

and

$$G_{nk}''(r)-kr^{-1}G_{nk}(r)=(M-E+\Sigma)F_{nk}(r), \tag{3}$$

where $\Delta(r)$ and $\Sigma(r)$ are

$$\Delta(r)=U_V(r)-U_S(r) \tag{4}$$

and

$$\Sigma(r)=U_V(r)+U_S(r).$$

We consider the case of exact spin symmetry for which $\Delta=0$ and $\Sigma(r)=V_{\text{spin-orbit}}(r)$. The spin-orbit potential $V_{\text{spin-orbit}}$ is

(5)

$$V_{spin-orbit}(r) = V_{so} \frac{e^{\frac{r-R}{a}}}{a(1+e^{\frac{r-R}{a}})^2 r},$$

$$V_{so} = -U_1(1-2ZA^{-1})U_{so}[j(j+1)-\ell(\ell+1)-s(s+1)]$$

where, Z and A are the atomic and the mass numbers, respectively, and $R = r_0 A^{1/3}$. The parameter 'a' refers to the thickness of the surface. The values used for the parameters are $r_0 = 1.2\,fm$ $U_1 = -0.075\,MeV$,

$U_{so} = 3.7516\,MeV$ and a=0.54fm. By composing Eqs. (2) and (3) and also considering $k = \ell$ Eq. (5), one gets

(6)

$$F''_{nk}(r) + \left[-\frac{\ell(\ell+1)}{r^2} + \frac{2(M+E)v_{so}}{r}\right.$$

$$\left.\frac{e^{\frac{r-R}{a}}}{a(1+e^{\frac{r-R}{a}})^2} + E^2 - M^2\right]F_{nk}(r) = 0.$$

The above equation cannot be solved, then we apply the Pekeris approximation [8] for the term $1/r^2$. To apply this approximation, we use the variable changing, $r = R(1+q)$ then we have

$$\frac{1}{r^2} = \frac{1}{R^2(1+q)^2} = \frac{1}{R^2}(1-2q+3q^2+...), \quad (7)$$

where the Taylor expansion is also used. On the other hand, using the Pekeris approximation, one can write

(8)

$$\frac{1}{r^2} = \frac{1}{R^2}(A_0 + \frac{A_1}{1+\exp(\frac{R}{a}q)} + \frac{A_2}{\left(1+\exp(\frac{R}{a}q)\right)^2}).$$

To determine A_0, A_1 and A_2 we expand the right side of Eq. (8) around $q = 0$ so that we obtain

(9)

$$\frac{1}{r^2} = \frac{1}{R^2}\{(A_0 + \frac{A_1}{2} + \frac{A_2}{4}) -$$

$$\frac{R}{4a}(A_1 + A_2)q + \frac{R^2 A_2}{16a^2}q^2 + \cdots\}.$$

A term-by-term comparison of equations (7) and (9) leads to the following quantities

(10)

$$A_0 = (R - 4a + 12\frac{a^2}{R})R^{-1},$$

$$A_1 = (8a - 48\frac{a^2}{R})R^{-1}, \quad A_2 = 48a^2 R^{-2}$$

By this approximation, Eq. (6) is converted to the following form

(11)

$$F''_{nk}(r) + \left[-\ell(\ell+1)R^{-2}(A_0 + \frac{A_1}{1+e^{\frac{r-R}{a}}} + \right.$$

$$\frac{A_2}{\left(1+e^{\frac{r-R}{a}}\right)^2}) + \frac{2(M+E)v_{so}}{r} \frac{e^{\frac{r-R}{a}}}{\left(1+e^{\frac{r-R}{a}}\right)^2}$$

$$\left. + E^2 - M^2\right]F_{nk}(r) = 0.$$

The presence of the term $1/r$ in the above equation still makes it difficult for our analytical solution. For this reason, we apply another approximation $1/r = e^{-r/a}/\omega_0$ where ω_0 is an adjustable parameter with a length dimension. It can be shown that taking $\omega_0 = 0.15\,fm$ and $a = 0.45\,fm$ leads to a reliable approximation. Using this approximation and introducing a new variable as $z = -e^{r/a}$, Eq. (11) is simplified to the following form

(12)

$$F''_{nk}(r) + \frac{(1-e^{\frac{R}{a}}z)}{z(1-e^{\frac{R}{a}}z)} F'_{nk}(r) + \frac{1}{z^2\left(1-e^{\frac{R}{a}}z\right)^2}$$

$$\left\{-\frac{\ell(\ell+1)}{a^2R^2}A_0 + \frac{2M(M-B)+(M-B)^2}{a^2}\right.$$

$$+\frac{e^{\frac{R}{a}}}{a^2}\left(\frac{\ell(\ell+1)}{R^2}(A_1+2A_0)+2(3M-B)(B-M)\right)z$$

$$-[\frac{e^{\frac{R}{a}}\ell(\ell+1)}{a^2R^2}\left(A_0+(A_2+A_1)e^{\frac{R}{a}}\right)$$

$$\left.+\frac{e^{\frac{R}{a}}(3M-B)}{a^2}\left(B-M-\frac{2v_{so}}{\omega_0}\right)]z^2\right\}F_{nk}(z)=0,$$

In the equations above, we set $E = -B + M$ B as the binding energy of baryon. Now, equation (12) is converted to a suitable form which can be solved by the NU (Nikiforov-Uvarov) approach in Ref [9, 10]. Ignoring the cumbersome and complicated detail of solution, our analytical result for the Dirac spinor $F_{nk}(r)$ is given by

$$F_{nk}(r) = N_0 \, (-e^{r/a})^{N_3}(1+e^{r/a})^{-N_3-\frac{N_4}{N_3}}$$
$$P_n^{(N_1-1,\frac{N_2}{N_3}-N_1-1)}(1+2e^{r/a}),$$

(13)

where, N_0 and P are the normalization constant and the Jacobi polynomials, respectively, and

(14)

$$N_1 = 1 + 2\sqrt{\beta_3} \, ,$$

$$N_2 = 2e^{\frac{R}{a}}(1+\sqrt{\beta_3})+2\sqrt{-\beta_2 e^{\frac{R}{a}}+\beta_3 e^{\frac{2R}{a}}+\frac{1}{4}e^{\frac{2R}{a}}+\beta_1}$$

$$N_3 = \sqrt{\beta_3} \, ,$$

$$N_4 = -\frac{1}{2}e^{\frac{R}{a}}-\sqrt{-\beta_2 e^{\frac{R}{a}}+\beta_3 e^{\frac{2R}{a}}+\frac{1}{4}e^{\frac{2R}{a}}+\beta_1}-e^{\frac{R}{a}}\sqrt{\beta_3}$$

Where

(15)

$$\beta_1 = \frac{e^{\frac{R}{a}}\ell(\ell+1)}{a^2R^2}\left(A_0+(A_2+A_1)e^{\frac{R}{a}}\right)+$$

$$\frac{e^{\frac{R}{a}}(3M-B)}{a^2}\left(B-M-\frac{2v_{so}}{\omega_0}\right),$$

$$\beta_2 = \frac{e^{\frac{R}{a}}}{a^2}\left(\frac{\ell(\ell+1)}{R^2}(A_1+2A_0)+2(3M-B)(B-M)\right),$$

$$\beta_3 = \frac{\ell(\ell+1)}{a^2R^2}A_0 + \frac{-2M(M-B)-(M-B)^2}{a^2}.$$

By substituting Eq. (13) into Eq. (3) one leads to the second component $G_{nk}(r)$ as

(16)

$$G_{nk}(r) = N_0 \frac{(-e^{-\frac{r}{a}})^{N_3}(1+e^{-\frac{r}{a}})^{-N_3-N_4}}{a(2M-E)}$$

$$\left\{-\left((n-1)+N_2\right)e^{r/a}\, P_{n-1}^{(N_1,N_2-N_1)}(1+2e^{-\frac{r}{a}})\right.$$

$$-\left(\frac{\ell a}{\omega_0}e^{-\frac{r}{a}}+(N_3+N_4)(1+e^{-\frac{r}{a}})^{-1}+N_3\right)$$

$$\left. P_n^{(N_1-1,N_2-N_1-1)}(1+2e^{-\frac{r}{a}})\right\}.$$

The magnetic moment is defined as $\vec{\mu} = \vec{\mu}_D + \vec{\mu}_a$ so that $\vec{\mu}_D$ and $\vec{\mu}_a$ are the Dirac and the anomalous magnetic moments, respectively. The anomalous magnetic moment μ_a one has [11]

(17)

$$\mu_a = 2\mu_B j\Omega_k \int r^2 dr\left[\frac{G^2_{nk}}{2\ell_k+1}+\frac{F^2_{nk}}{2\ell_{-k}+1}\right],$$

Table 1. Dirac magnetic moments of lambda-hypernuclei $^{13}_{\Lambda}\text{C}$, $^{17}_{\Lambda}\text{O}$ and $^{41}_{\Lambda}\text{Ca}$ ($\mu_D (10^{-4}\text{n.m})$).

Hyper nuclei	$1s_{1/2}$	Ref [11]	$1p_{3/2}$	Ref [11]	$1p_{1/2}$	Ref [11]
$^{13}_{\Lambda}\text{C}$	-390	-395	-329	-322	-224	-215
$^{17}_{\Lambda}\text{O}$	-489	-483	-513	-503	-378	-369
$^{41}_{\Lambda}\text{Ca}$	-711	-701	-1046	-1065	-556	-546

where $\Omega_k = 1(-1)$, $\ell_k = -k-1(k)$ for $k < 0 (k > 0)$. Also, for the Dirac magnetic moment, μ_D one has [12]

$$\mu_D = -\frac{j}{2}\int r^2 \, dr \, (G^2_{nk}(r) - F^2_{nk}(r) - \frac{\Omega_k}{2\ell_k+1}G^2_{nk}(r) - \frac{\Omega_k}{2\ell_{-k}+1}F^2_{nk}(r))B_f(r) \tag{18}$$

where the reduction factor $B_f(r)$ is defined as

$$B_f(r) = \frac{g_{\Lambda\nu}}{g_{N\nu}} \times \frac{M_N}{M_\Lambda}\left[1 + \frac{\sqrt{\left(\frac{3\pi^2}{2}\rho_N(r)\right)^{2/3} + M_N^2}}{\lambda_{N\nu}\rho_N(r)}\right]^{-1} \tag{19}$$

The equation above stands for the mass of Λ-hyperon and nucleon, respectively, $g_{\Lambda\nu}/g_{N\nu} = 2/3$ and $\lambda_{N\nu} = g^2_{N\nu}/m^2_\nu$ in which $g_{N\nu} = 13$ [12]. The density distribution of the nuclear core $\rho_N(r)$ is given by [13]

$$\rho_N(r) = \rho_0\left\{1 + e^{(r-R)/a}\right\}^{-1} \tag{20}$$

where $\rho_0 = 0.14\text{fm}^{-3}$. We use F(r) and G(r) to calculate the magnetic moment. In the following, we present our numerical analysis of the magnetic moment.

III. Results and Discussion

In Table 1, our results are compared with the ones presented in Ref. [11]. It should be noted that in Ref. [11] the effects of core polarization and tensor coupling on the magnetic moments are studied through the Dirac equation in the presence of tensor potential. It is shown that the inclusion of a tensor coupling suppresses the effect of core polarization on magnetic moments. Although, since the hyperon wave functions are not sensitive to the tensor potential, the magnetic moments with or without tensor potential are almost the same. Therefore, the small differences between both results in Table 1 directly consequence of the coupling chosen for the lambda hyperon. In Table 2, we listed our theoretical results for the total magnetic moments of ground and excited Λ-hypernuclei, i.e. lambda hyperons in the states 1s1/2, 1p3/2 and 1p1/2. They are also compared with the values obtained by Ref [11]. As it is seen, there are good agreements between both results; however, the original differences arises from the

Table 2. Magnetic moments of ground and excited lambda

	$1s_{1/2}$		$1p_{3/2}$
Hyper nuclei	This work	Ref [11]	This work

tensor coupling effect, which is completely ignored in our work due to the complexity of the Dirac equation in the presence of the tensor coupling term. Note that, in hypernuclei, the tensor coupling is essential to reproduce the weak spin-orbit splitting in Lambda hyperon [14].

IV. Conclusions

Among all, the magnetic moment of Λ-hypernuclei is one of the most important observables related to hypernuclear physics. In this regard, it provides possible tests of relativistic models of nuclei and does also have an important implication for theoretical predictions of hadronic properties in nuclear matter. Since long ago, there has been an increasing interest in relativistic effects arising in models of the nucleus based on the Dirac equation with strong scalar and vector potentials. Although relativistic and nonrelativistic models yield similar results for the isoscalar magnetic moments, there is a fundamental difference between these approaches [15]. In the relativistic picture, the Schmidt values are obtained from two canceling effects, while they arise directly in the nonrelativistic shell model. Thus, finding cases for

which these underlying theories have other predictions is desirable. In this work, in a relativistic approach, we analytically determined the magnetic moments of several lambda hypernuclei, considering the spin-orbital potential. Using the NU method, a first effort is made to calculate the wave function analytically. The magnetic moments are computed for the ground and excited states of several lambda hypernuclei.

References

1. T. Yamazaki, *Hadron nucleus bound states*, Phys. Lett. B. **160**, 227 (1985).

2. X. Liu et al. *First investigation of hypernuclei in reactions via analysis of emitted bremsstrahlung photon*, PRC. **99**, 064614 (2019).

3. M. Mehrotra and I. *Mehrotra, Deviation from the Schmidt value for the magnetic moments of mirror hypernuclei*, Nucl. Phys. **63**, 122 (2018).

4. H. Y. Sang et al. *Magnetic moments of Lambda hypernuclei within the time-odd triaxial relativistic mean-field approach*, PRC. **88**, 064304 (2013).

5. K. Saito, M. Oka and T. Suzuki, *Roles of quark degrees of freedom in hypernuclei*, Nucl. Phys. A. **95**, 625 (1997).

6. J. Mare and J. Zofka, *Magnetoresistance of high temperature superconductors*, Physics Letters B. **249**, 181(1990).

7. O. Hashimoto and H. Tamura, *Spectroscopy of Lambda hypernuclei, Prog*. Part. Nucl. Phys. **57**, 564 (2006).

8. C. L. Pekeris, *The Rotation-Vibration Coupling in Diatomic Molecules*, Physical Review. **4** 598 (1934).

9. S. M. Moosavi Nejad and A. Armat, *Relativistic excited state binding energies and RMS radii of Λ-hypernuclei*, Modern Physics Letters A. **33**, 1850022 (2018).

10. A. Boumali, L. Chetouani and H. Hassanabadi, *On the eigensolutions of one-dimensional Duffin-Kemmer-Petiau oscillator*, Can. J. Phys. **91**, 1 (2013).

11. M. Chemtob and M.Rho, *Meson-exchange currents and the 3 He magnetic form factor*, Nucl. Phys. A. **163**, 1 (1971).

12. Jiang-Ming et.al, *Core Polarization and tensor coupling effects on magnetic moments of hypernuclei*, Chin. Phys. Lett. **25**, 1629 (2008).

13. R. Brockmann, *Relativistic single particle motion and spin-orbit coupling in nuclei and hypernuclei*, Nuclear Physics A. **355**, 365 (1981).

14. J. F. Mathiot, *Electromagnetic meson-exchange currents at the nucleon mass scale*, Phys. Rep. **173**, 63 (1989).

15. J. Cohen and R. J. Furnstahl, *Hypernuclear currents in a relativistic mean-field theory*, Phys. Rev. C. **35**, 2231 (1987).

16. https://www.standardmethods.org/doi/10.2105/SMW, technology. Butterworth-Heinemann (2017).